# ESTIMATING THE SIZE OF A FISH POPULATION USING VESSEL SURVEY DATA

JOHN RADOVICH
Marine Resources Operations, California Department of Fish and Game

## INTRODUCTION

The methods described in this paper were developed to enable personnel of the Department of Fish and Game to estimate the population numbers of the Pacific sardine. The basic data required are the results of research surveys, e.g., those described in Radovich (1952), and standard age frequency data from the commercial fishery. A major prerequisite is that the species in question be vulnerable to the survey technique one year prior to the time it is recruited to the fishery. The technique has been presented orally to several research groups subsequent to its development in 1952, and the interest generated was the stimulus for the present brief report.

Originally, the symbols used for mortality and survival rates and for rates of exploitation were the same as those of Ricker (1948). However, since then, Ricker (1958) revised his symbols and, in addition, an international standard terminology for fishery dynamics has been proposed (Holt *et al.*, 1959). Because most fisheries biologists on the west coast of North America are familiar with Ricker's notation, I have used his revised notation (Ricker, 1958) whenever practicable. For those who prefer the standard terminology of Holt *et al.* (1959), I have placed a glossary of the terms I have used at the end of this paper and have compared them with the other two systems (Table 4).

## REASON FOR SURVEYS

The size or availability of a population of pelagic fish is most difficult, if not impossible, to determine from catch data alone when the species is not distributed uniformly throughout its range, when the proportion of fish in different parts of its range varies greatly in different years, and when a fishery exists in only a small part of the species' range. Under these conditions, it is necessary to extend sampling for population density estimates beyond the range of the fishery to include the whole range of the fish.

## METHOD

In most types of fish surveys designed to estimate relative abundance or population size, the statistic most readily obtained is the average catch-per-effort of the sampling gear used. The average catch-per-effort value from a given region is a relative density value and can be expressed as

$$\frac{\Sigma c_s}{\Sigma E} = D$$

wnere

$c_s$ = catch by sampling gear,

E = sampling effort,

and

D = relative density.

This relative density value applies only to the region covered and sampled during the survey. One may think of this density value as being directly proportional to numbers of fish per square mile.

If D were the true density, represented by numbers of fish per square mile, the product of D and the area in square miles would be the total population size. However, in this case the product of the relative density D and the area (of the region sampled) is proportional to the true density and may be regarded as relative abundance when compared to similar values for other areas or to values of the same area at different times. This may be expressed as

$$DA = R$$

where

A = area,

and

R = relative abundance.

The relative abundance of a greater region is simply the sum of the relative abundances of the smaller regions. The relative abundance of the population is the sum of the relative abundances of all regions making up the range of the fish.

The relative abundance of the total population at a given time, $t$, divided into the relative abundance one year later, $t+1$, excluding additions to the population between the time intervals, gives the annual survival rate of the same combined year-classes, or

$$\frac{R_{t+1}}{R_t} = s$$

where

$s$ = annual survival rate.

Within any given area, if the area is constant, the relative densities at two given times are proportional to relative abundance, and the survival rate is the ratio of the density at $t+1$ to the density at $t$, or

$$\frac{R_{t+1}}{R_t} = \frac{(D_{t+1})A_t}{D_t A_t} = \frac{D_{t+1}}{D_t} = s.$$

The survival rate thus calculated is some fraction that is less than unity, and the total mortality rate equals the difference between the survival rate and unity, or

$$a = 1 - s$$

where

$a$ = total annual mortality rate from all causes.

The following relationships and definitions are after Ricker (*op. cit.*):

$$a = m + n - mn$$

where

$m$ = annual fishing mortality rate if no natural mortality occurred,
$n$ = annual natural mortality rate if no fishing mortality occurred,

and

$a$ = total annual mortality rate.

By transposition of the above equation

$$m = \frac{a - n}{1 - n}.$$

In addition, Ricker (*op. cit.*) gives the following relationships:

$$a = u + v,$$

$$i = p + q,$$

and

$$\frac{i}{a} = \frac{p}{u}$$

or

$$u = \frac{ap}{i}$$

where

$u$ = rate of exploitation (fishing mortality rate when natural mortality occurs),
$v$ = expectation of natural death (natural mortality rate when there is fishing mortality),
$p$ = instantaneous rate of fishing mortality ($p = -\log_e(1 - m)$),
$q$ = instantaneous rate of natural mortality ($q = -\log_e(1 - n)$),

and

$i$ = instantaneous rate of total mortality ($i = -\log_e(1 - a)$).

The rate of exploitation $u$ is the fraction of the total population N that is actually caught, or

$$u = \frac{C}{N}$$

where

C = commercial catch in numbers.

## A HYPOTHETICAL EXAMPLE

Let us assume two hypothetical populations, as shown in Figure 1, in which the range of a fish population is divided into three regions: A, B, and C. In each situation the entire range of the population is surveyed, yielding 2,000 fish per standard-effort-

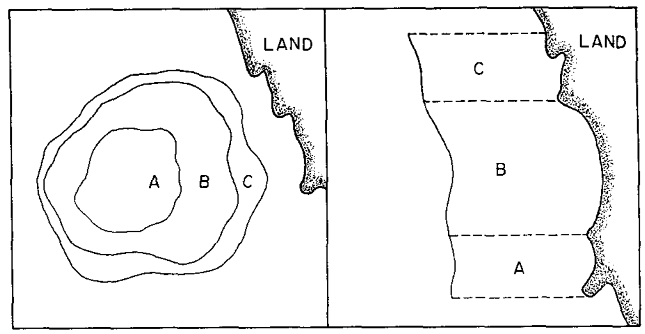


FIGURE 1. Two hypothetical fish populations, each divided into three regions: A, B and C. The population on the left is associated with an offshore bank; the one on the right is distributed along the coast. The method described in the text is applicable to either situation.

unit in region A, broken down by ages as 1,000 two-year-olds, 500 three-year-olds, 300 four-year-olds, 150 five-year-olds, and 50 six-year-olds. Region B yielded 400 twos, 200 threes, 125 fours, 75 fives and 25 sixes, for a total of 825 fish per unit of effort. Region C yielded 100 twos, 50 threes, and 20 fours and older, for a 170 fish total. The data may be summarized as in Table 1.

Let us assume that region A encompassed 300 square miles of habitat for the species being considered, region B encompassed 650 square miles, and region C, 325 square miles. Since catch-per-effort values of Table 1 are relative densities, when multiplied by square miles they become relative abundance values (DA=R). Thus, a comparable table of relative abundance may be constructed (Table 2).

TABLE 1
**Hypothetical Catch-per-effort from Survey by Age**

| Region | Twos | Threes | Fours and Older | Total |
|---|---|---|---|---|
| A | 1,000 | 500 | 500 | 2,000 |
| B | 400 | 200 | 225 | 825 |
| C | 100 | 50 | 20 | 170 |

From Table 2, total (x) shows the relative abundance of fish of each age with respect to the others and total (y) shows the relative abundance of fish of all ages in each region. Let us assume that this

TABLE 2

**Relative Abundance by Age (Hypothetical Example)**

| Region | Twos | Threes | Fours and Older | Total (y) |
|---|---|---|---|---|
| A | 300,000 | 150,000 | 150,000 | 600,000 |
| B | 260,000 | 130,000 | 144,250 | 534,250 |
| C | 32,500 | 16,250 | 6,500 | 55,250 |
| Total (x) | 592,500 | 296,250 | 300,750 | 1,189,500 |

TABLE 3

**Relative Abundance by Age (Hypothetical Example)**

| Year | Twos | Threes | Fours and Older | Total |
|---|---|---|---|---|
| 1st (Total (x) from Table 2) | 592,500 | 296,250 | 300,750 | 1,189,500 |
| 2nd | 473,000 | 362,000 | 240,000 | 1,255,000 |

survey is repeated at the same time in the same manner during the next year; the results are as indicated in Table 3. We can then calculate the survival rate of fish between ages two and three as follows:

$$\frac{362{,}000}{592{,}500}\cdot$$

The annual survival rate of fish older than age two is

$$\frac{240{,}000}{296{,}250 + 300{,}750}$$

or

$$\frac{R_{t+1}}{R_t} = s.$$

The survival rates thus calculated are 0.611 between ages two and three, and 0.402 for the older fish. Since $a = 1 - s$, the total annual mortality rate for fish between ages two and three is $a = 1 - 0.611 = 0.389$, and for fish older than two years is $a = 1 - 0.402 = 0.598$.

If for some reason (gear selection, fish habits, or other factors) the fish are not available to the fishery before they are three years old, and provided that we can assume the natural mortality rate between ages two and three represents the annual natural mortality rate of older fish,

then

$$n = 0.389$$

and

$$a = 0.598,$$

and since

$$m = \frac{a - n}{1 - n},$$

then

$$m = \frac{0.598 - 0.389}{1 - 0.389} = \frac{0.209}{0.611}$$

or

$$m = 0.342.$$

From Ricker's tables (1958), $i = 0.91125$ corresponds to $a = 0.598$, and since $p$ bears the same relationship to $m$ as $i$ does to $a$, one can find a $p$ value corresponding to $m = 0.342$ by locating 0.342 on the $a$ column and reading $p = 0.4185$ from the $i$ column. The rate of exploitation of $u$ is then calculated as follows:

$$u = \frac{ap}{i} = \frac{(0.598)(0.4185)}{0.91125},$$

or

$$u = 0.2746,$$

and since

$$N = \frac{C}{u}$$

the population can be calculated if the catch data (numbers of fish taken by the commercial catch) are available. If, for example, the catch was 500,000 fish in the season between surveys, then

$$N = \frac{C}{u} = \frac{500{,}000}{0.2746}$$

or

N = 1,820,000 fish that were three years old and older. The total population of fish three years old and older would be 1,820,000 (or 1.82 million) at the beginning of the first year. The total population at $t$ of fish two years old and older, $N_{2+}$, bears the same relationship to the population of fish three years old and older, $N_{3+}$, as the relative abundance of fish two and older, $R_{2+}$, does to the relative abundance of fish three and older, $R_{3+}$, or at $t$

$$\frac{N_{2+}}{N_{3+}} = \frac{R_{2+}}{R_{3+}}$$

or

$$N_{2+} = \frac{(R_{2+})(N_{3+})}{R_{3+}} \ .$$

The population in the first year $t$, including two-year-old fish, $N_{2+}$, would then be

$$N_{2+} = \frac{(1{,}189{,}500)(1{,}820{,}000)}{296{,}250 + 300{,}750}$$

or

$$N_{2+} = 3{,}626{,}281$$

or 3.63 million fish are past their second year of life. The population of age two and older fish in the second year, $t+1$, may be calculated by the equation

$$N_{t+1} = \frac{(R_{t+1})(N_t)}{R_t}$$

where

$N_t$ = total population in the first year ($N_{2+}$ at $t$),
$N_{t+1}$ = total population in the second year,
$R_t$ = total relative abundance in the first year, and
$R_{t+1}$ = total relative abundance in the second year.

Substituting into the equation we get

$$N_{t+1} = \frac{(1{,}255{,}000)(3{,}626{,}281)}{1{,}189{,}500}$$

and

$N_{t+1}$ = 3.83 million fish two years old and older in the population at time $t+1$.

## ALTERNATE METHOD

An alternate formula for calculating population size more directly, by using common logarithms, can be derived in the following manner:

since

$$s = e^{-i} = e^{-(p+q)} \ ,$$

the survival rate of younger fish that are not vulnerable to the fishery $s_y$ can be expressed as

$$s_y = e^{-q} \ .$$

It then follows that

$$q = -\log_e s_y$$

and

$$p = \log_e s_y - \log_e s \ .$$

By combining the equations

$$N = \frac{C}{u}$$

and

$$u = \frac{ap}{i}$$

and substituting for $i$, $a$ and $p$, we get

$$N = \frac{C(-\log_e s)}{(1 - s)(\log_e s_y - \log_e s)}$$

or

$$N = \frac{C(\log_e s)}{(s - 1)\log_e (s_y/s)} \ .$$

Finally, since the natural or Napierian logarithm of a number is equal to that number's common logarithm times a constant, we may substitute the equivalent common logarithms and their constants into the equation. The constants cancel out and we end up with

$$N = \frac{C(\log_{10} s)}{(s - 1)\log_{10} (s_y/s)} \ .$$

In the hypothetical example given previously, the survival rate for the younger (non-vulnerable) fish was 0.611, and for the older (vulnerable) fish, it was 0.402. By substituting these values, and the value for the number of fish in the catch (500,000), into the formula, we may solve for the population size directly:

$$N = \frac{500{,}000 \ \log_{10} 0.402}{(0.402 - 1) \ \log_{10} \left(\frac{0.611}{0.402}\right)} \ .$$

We need only to substitute the values for the common logarithms and complete the calculations to find

$$N = 1.82 \text{ million fish,}$$

which is the same value we previously obtained, for the population size at the beginning of the first year.

## ASSUMPTIONS

The accuracy of this method in approximating a real population is dependent upon the degree to which the equations describe actual phenomena. A perfect fit of this method into a real situation implies that a number of assumptions will be met. Following are some of the more apparent ones:

1. The entire population is an entity that will not change by movement out of or into the range of the survey, *i.e.* the survey encompasses the range of the population.
2. Natural mortality is distributed evenly throughout the year.
3. Fishing mortality is distributed evenly throughout the year. This condition may not be met in an actual fishery. However, if the "fishing year" is adjusted so that the fishing period is in the middle of that year, the error will be minimized.

4. The survey randomly samples the population and its catch-per-effort is proportional to the true population density.
5. The observed natural mortality rate of the young fish is approximately the same for older fish in the population.
6. Other mortality caused by fishing, in addition to the amount landed, is negligible.
7. Age reading is accurate.
8. Catch figures are reliable and the conversion from weight to numbers is valid.

TABLE 4

**GLOSSARY OF SYMBOLS AND DEFINITIONS**

Compared with those of Ricker (1958) and Holt *et al.* (1959)

| Symbols Used | Definition | Ricker (1958) | Holt *et al.* (1959) |
|---|---|---|---|
| $c_s$ | Catch by sampling gear | | |
| E | Sampling effort | | |
| D | Relative density $\left(D = \frac{\Sigma c_s}{\Sigma E}\right)$ | | |
| A | Area sampled or covered by survey | | |
| R | Relative abundance in area covered (R = DA) | | |
| $t$ | Time (a subscript indicating a specific time) | $t$ | $t$ |
| $m$ | Annual fishing mortality rate if no natural mortality occurred ($m = 1 - e^{-p}$) | $m$ | $1 - e^{-F}$ |
| $n$ | Annual natural mortality rate if no fishing mortality occurred ($n = 1 - e^{-q}$) | $n$ | $1 - e^{-M}$ |
| $u$ | Annual rate of exploitation (fishing mortality rate when natural mortality occurs; $u = ap/i$) | $u$ | $E$; $(1 - S)\frac{F}{Z}$; unconditional fishing mortality rate |
| $v$ | Annual expectation of natural death (natural mortality rate when there is fishing mortality; $v = aq/i$) | $v$ | $D$; $(1 - S)\frac{M}{Z}$; unconditional natural mortality rate |
| $a$ | Total annual mortality rate from all causes ($a = 1 - s$; $a = u + v$; $a = m + n - mn$) | $a$ | $(1 - S)$; $1 - e^{-Z}$ |
| $p$ | Instantaneous rate of fishing mortality ($p = -\log_e(1 - m)$) | $p$ | $F$; fishing mortality coefficient |
| $q$ | Instantaneous rate of natural mortality ($q = -\log_e(1 - n)$) | $q$ | $M$; natural mortality coefficient |
| $i$ | Instantaneous rate of total mortality ($i = -\log_e(1 - a)$; $i = p + q$) | $i$ | $Z$; $-dN/Ndt = F + M$; total mortality coefficient |
| $s$ | Annual survival rate ($s = e^{-i}$) | $s$ | $S$; $e^{-Z}$; fraction surviving |
| $s_y$ | Annual survival rate for fish which are not yet vulnerable to the fishery | | |
| C | Total catch by the fishery in numbers | C | $C$ |
| N | Number of fish in the population | N | $N$ |

## REFERENCES


Holt, S. J., J. A. Gulland, C. Taylor and S. Kurita. 1959. A standard terminology and notation for fishery dynamics. Inter. Cons. Explor. Mer., Jour., vol. **24**, no. 2, pp. 239-242.

Radovich, John. 1952. Report on the young sardine, *Sardinops caerulea*, survey in California and Mexican waters, 1950 and 1951. Calif. Dept. Fish and Game, Fish. Bull. 87, pp. 31-63.

Ricker, W. E. 1948. Methods of estimating vital statistics of fish populations. Indiana Univ., Publ., Sci. ser. no. 15, 101 pp.

—. 1958. Handbook of computations for biological statistics of fish populations. Fish. Res. Bd. Canada, Bull. 119, 300 pp.